ANNOTATION
to the dissertation submitted for the degree of Doctor of Philosophy (PhD) in the specialty "6D061100 – Physics and Astronomy"

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GRAVITATIONAL FIELDS OF COMPACT OBJECTS IN GENERAL RELATIVITY

General Characteristics of the Research

The class of compact objects typically includes planet-like bodies, white dwarfs, neutron stars, other exotic dense stars, and black holes. It is known that Newtonian gravity is sufficient to describe the gravitational field of traditional astrophysical objects. However, the discovery of exotic compact objects such as quasars and pulsars, as well as the possibility of gravitational collapse leading to black holes, indicates the growing importance of relativity theory in astronomy and astrophysics. Furthermore, advances in space exploration and the development of modern measurement technologies have necessitated the inclusion of relativistic effects even within the Solar System.

One of the major tasks of modern relativistic gravity is the description of the Universe using Einstein's theory of relativity, particularly the study of gravitational fields around celestial bodies such as stars and black holes, and the explanation of their dynamic properties.

General Theory of Relativity is Einstein’s fundamental theory that describes gravity through the curvature of spacetime. It extends Newtonian mechanics and provides a framework to explain astrophysical phenomena in strong gravitational fields. General Theory of Relativity is of great importance not only in theoretical physics but also in astrophysics, celestial mechanics, cosmology, the physics of compact objects, and gravitational waves.

This dissertation is devoted to describing the gravitational field of compact objects, including neutron stars and black holes, within the framework of general theory of relativity. It examines the conditions for the formation of black holes, their physical characteristics, and gravitational effects. The dissertation work also analyzes the agreement of observational data on supermassive black holes with general theory of relativity predictions, explores the domain of applicability of general theory of relativity solutions, and presents a comparative analysis with alternative models. This research aims to provide a deeper understanding of the nature of compact objects based on theoretical models and modern observational data.

Relevance of the Study

One of the key areas of modern physics and astronomy is the study of gravitational fields around celestial bodies such as stars and black holes within the framework of general theory of relativity. General theory of relativity has substantial practical applications: for example, in global positioning systems (GPS) or in calculating the trajectories of spacecraft such as Voyager and New Horizons,

corrections based on this theory are required. It is also impossible to accurately describe neutron stars, X-ray binary systems, or objects in galactic centers without taking general theory of relativity into account.

Black holes are among the primary objects of research in modern astrophysics. Before the advent of general theory of relativity, hypothetical objects like "black stars" were proposed. However, since 1967, with the discovery of pulsars, black hole candidates in binary star systems, active galactic nuclei, quasars, and blazars, black holes have been recognized as real astrophysical objects. The compact bodies observed in dense binary systems did not match any known neutron stars or white dwarfs at that time. Moreover, objects with masses of millions, and in some cases billions, of solar masses have been discovered in the centers of galaxies, the nature of which can only be explained by the existence of supermassive black holes.

Since 2016, following the detection of gravitational waves, interest in the physics of stellar-mass black holes has significantly increased. This phenomenon, predicted by Einstein, was observed using highly sensitive laser interferometers such as LIGO (Laser Interferometer Gravitational-Wave Observatory) and Virgo (Interferometric Gravitational-Wave Detector Observatory), along with the development of new detection and data analysis methods.

Obtaining the image of the shadow of supermassive black holes at the centers of the M87 galaxy and the Milky Way became a major achievement of global scientific collaboration. This research confirmed the predictions of general theory of relativity and proved the validity of the theory.

Currently, although alternative black hole solutions such as Schwarzschild, Reissner-Nordström, and Kerr are being proposed, observational precision is still insufficient to distinguish between them.

It is worth noting that in 2017, Rainer Weiss, Barry Barish, and Kip Thorne were awarded the Nobel Prize for their contributions to the detection of gravitational waves. In 2020, Roger Penrose, Reinhard Genzel, and Andrea Ghez received the Nobel Prize for discoveries related to black holes.

Understanding the step-by-step history of the origin of the Universe is also an achievement of general theory of relativity and modern astrophysics. From the Big Bang (13.8 billion years ago), the formation of the first elements, the separation of the cosmic microwave background, the birth of the first stars and galaxies, and the formation of the Solar System – all these processes are based on physical laws. Today, the expansion of the Universe continues.

However, formation mechanism of supermassive black holes remains unclear. There are various hypotheses: the direct collapse of primordial gas clouds, mergers of stellar-mass black holes, or accretion of matter by remnants of the first stars. In 2025, the event AT2024tvd was recorded, during which astronomers observed, for the first time, a wandering supermassive black holes consuming a star on the outskirts of a galaxy. This shows that black holes can exist not only in galactic centers but also in other regions. Research on black holes will continue in the future.

Thus, the origin, properties, and characteristics of black holes within the framework of general theory of relativity are one of the central areas of modern astrophysics. Therefore, the topic, purpose, and objectives of this dissertation fully correspond to the specialty “Physics and Astronomy.”

Research Objective

Study of the gravitational fields of black holes and neutron stars based on general theory of relativity.

Research Objects

Strong gravitational fields, black holes and neutron stars, Einstein's field equations. Motion of test particles.

Research Subject

Exact and approximate solutions of Einstein's field equations. Adiabatic theory of body motion in general theory of relativity. Mechanism of the formation of supermassive black holes.

Research Methods

The following theoretical and numerical methods were used in this study:

- Analytical and numerical solutions of Einstein's field equations;
- Methods of differential geometry and tensor analysis;
- Lagrangian and Hamiltonian formalism, perturbative method;
- Numerical modeling.

Research Tasks

To achieve the aim of this work, the following objectives were set:

1. To find the connection between the approximate Erez-Rosen metric and the external static Hartle-Thorne metric.
2. To study the motion of bodies in general theory of relativity using the adiabatic theory.
3. To investigate the formation of supermassive black holes resulting from the gravitational collapse of fermionic dark matter.

Scientific Novelty and Contribution

In this work, for the first time:

1. A connection was established between the approximate Erez-Rosen metric and the external static Hartle-Thorne solution without applying the Zipoy–Voorhees transformation, in the approximation of Q and M^2.
2. The description of test particle motion within the adiabatic theory allowed the derivation of a formula for the perihelion precession for the Hartle-Thorne and Kerr metrics in the $1/c^2$ approximation. The influence of the central body's rotation and deformation on the test particle’s trajectory was demonstrated. It was also proven that the resulting formula complies with the principle of superposition of relativistic effects.
3. A new mechanism for the formation of supermassive black holes was proposed. According to this mechanism, supermassive black holes form as a result of the gravitational collapse of dark matter in the centers of galaxies. It was shown that such black holes subsequently increase their mass through accretion.

Scientific Statements for Defense

1. The connection between the approximate Erez-Rosen metric and the static Hartle-Thorne metric in the approximation of Q and M^2 does not require the Zipoy-Voorhees transformation, unlike in the Q approximation, where the connection between the two metrics can only be established through the Zipoy-Voorhees transformation.

2. The effects arising from the mass, angular momentum, and quadrupole moment of the gravitational field source are described by the principle of superposition of relativistic effects. The influence of the Sun's angular and quadrupole moments on the perihelion shift of the planets is negligible.

3. Primordial black holes at the galaxies are formed as a result of the gravitational collapse of dense fermionic dark matter cores. This process is significantly faster than traditional stellar mechanisms and contributes to the formation of supermassive black holes.

Theoretical and Practical Significance

The research results contribute to the development of theoretical and numerical methods in astrophysics, compact object physics, and cosmology. Moreover, the obtained theoretical results help explain the sources of X-ray pulsars and gamma-ray emissions.

Reliability and Validation of Results

The obtained results are consistent with observational data. The credibility of the dissertation is confirmed by articles published in high-impact international journals and in publications recommended by the Committee for Quality Assurance in the Sphere of Science and Higher Education of the Ministry of Science and Higher Education of the Republic of Kazakhstan.

Author's Personal Contribution

The author independently carried out the entire research process from beginning to end, including the literature review and all numerical calculations, in consultation with the scientific supervisors. She took an active role in selecting the research methods and jointly analyzed the obtained results with her supervisors.

Thesis Approval and Dissemination

During the preparation of the dissertation, 8 printed publications were published. Among them, 2 articles were published in reputable journals recommended by the Committee for Quality Assurance in the Field of Science and Higher Education of the Ministry of Science and Higher Education of the Republic of Kazakhstan – *International Journal of Mathematics and Physics* and *Proceedings of the National Academy of Sciences of the Republic of Kazakhstan. Series of Physical and Mathematical Sciences*. 3 articles were published in international journals with high impact factors indexed in Web of Science and Scopus databases. Additionally, 3 abstracts were presented at international conferences.

This dissertation makes a significant contribution to the study of gravitational fields of compact objects within the framework of general theory of relativity.

Articles with high impact factors published in journals indexed in the Thomson Reuters or Scopus international scientific databases

1. Boshkayev K., Quevedo H., Nurbakyt G., Malybayev A., Urazalina A. The Erez-Rosen Solution versus the Hartle-Thorne Solution // Symmetry – 2019. – Vol.11(10). – art. № 1324. *(***WoS (JCR):** Impact Factor (2019) = 2.645, Q2. **Scopus (CiteScore):** 54%*).*

2. Arguelles C. R., Boshkayev K., Krut A., Nurbakyt G., Rueda J. A., Ruffini R., Uribe-Suarez J. D., Yunis R. On the growth of supermassive black holes formed from the gravitational collapse of fermionic dark matter cores // Monthly Notices of the Royal Astronomical Society, (MNRAS) – 2023. – Vol. 523(2). – P. 2209-2218. *(***WoS (JCR):** Impact Factor (2022) = 4.8, Q1. **Scopus (CiteScore):** 85%*).*

3. Boshkayev K., Nurbakyt G., Quevedo H., Suliyeva G., Taukenova A. S., Tlemissov A., Tlemissova Zh., Urazalina A., Dalelkhankyzy A., Stuchlik Z., Beissen N., Gumarova Sh. "Adiabatic theory in Kerr spacetimes" // General Relativity and Quantum Cosmology. – 2024. – Vol. 56(5). – art. №. 67 *(***WoS (JCR):** IF (2022) = 2.0, Q2. **Scopus (CiteScore):** 73%*).*

Publications in journals recommended by the Committee for Quality Assurance in the Field of Science and Higher Education of the Ministry of Science and Higher Education of the Republic of Kazakhstan

1. Boshkayev K., Malybayev A., Quevedo H., Nurbakyt G., Taukenova A., Urazalina A. The correspondence of the Erez-Rosen solution with the Hartle-Thorne solution in the limiting case of ~Q and ~M^2//News of the National Academy of science of the Republic of Kazakhstan: Physico-Mathematical Series, – 2020. – Vol. 5, no.333 – P. 19-27.

2. Suliyeva G. B., Boshkayev K., Nurbakyt G., Quevedo H., Taukenova A. S., Tlemissov A. T., Tlemissova Zh. A., Urazalina A. Adiabatic theory of motion of bodies in the Hartle-Thorne spacetime // International Journal of Mathematics and Physics. – 2022. – Vol. 13 (1). – P. 82-90.

Publications in the proceedings of international and domestic scientific conferences

1. Boshkayev K.A., Nurbakyt G. Correspondence of the Erez-Rosen metric with the q-metric // International Scientific Conference of students and Young scientists "Farabi Alemi" of Al-Farabi Kazakh National University, Almaty – 2019. – P. 13.

2. Boshkayev K.A., Zhumakhanova G., Nurbakyt G. Erez-Rosen metric versus q-metric // The 6th International Workshop of Nuclear Physics, Nuclear Astrophysics and Cosmic Rays Almaty, Kazakhstan, – 2019. – P. 58.

3. Нұрбақыт Г. Қонысбаев Т., Курманов Е. Квазипериодты тербелістер арқылы шағын нысандардың негізгі параметрлерін анықтау // «Фараби әлемі» атты студенттер мен жас ғалымдардың халықаралық ғылыми конференция материалдары, Алматы, Қазақстан. – 2022. – Б. 54.

Dissertation Structure and Scope

The dissertation consists of an introduction, 4 chapters, a conclusion, and a list of references. The work includes 316 formulas, 2 tables, 11 figures, a bibliography of 191 sources, and comprises 108 pages.